A splitter theorem for internally 4-connected GRAPHS (and binary matroids)

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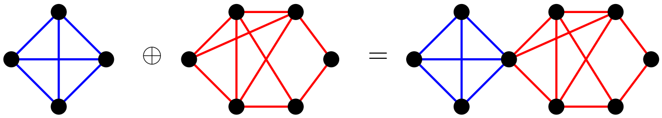
Connectivity



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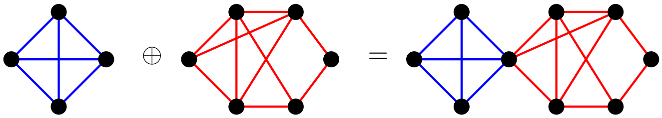

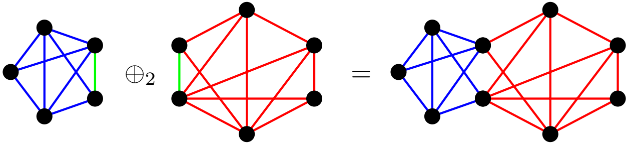

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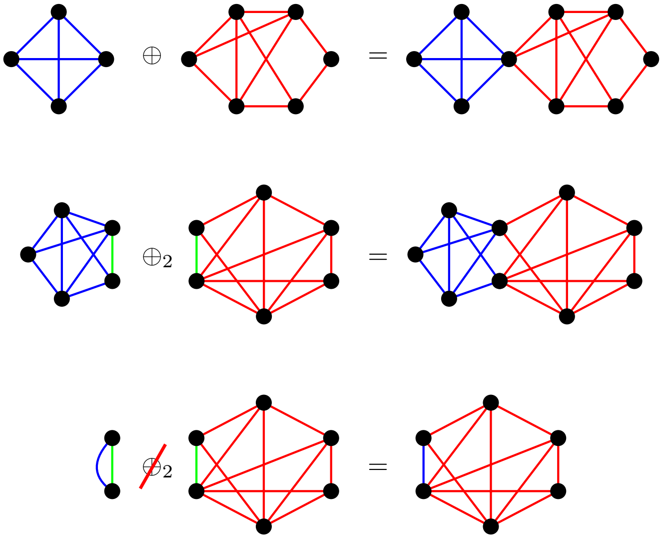
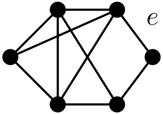


Chain theorems

Theorem (Tutte, 1966)

G 2-connected $\Longrightarrow$ $G \backslash e$ *or* G/e *is 2-connected for all* $e \in E(G)$



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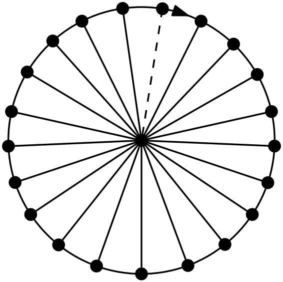
G is a wheel



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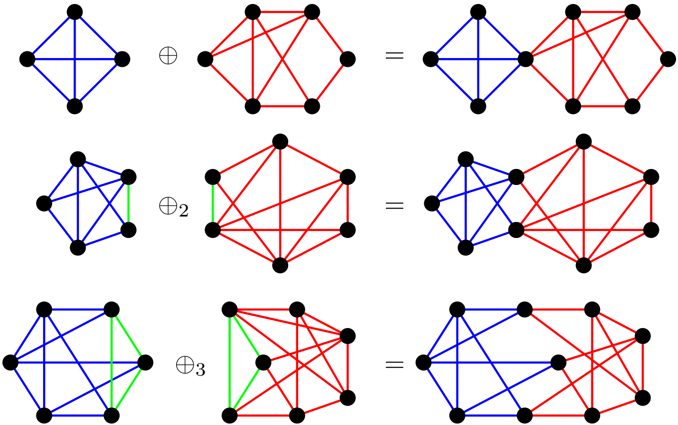

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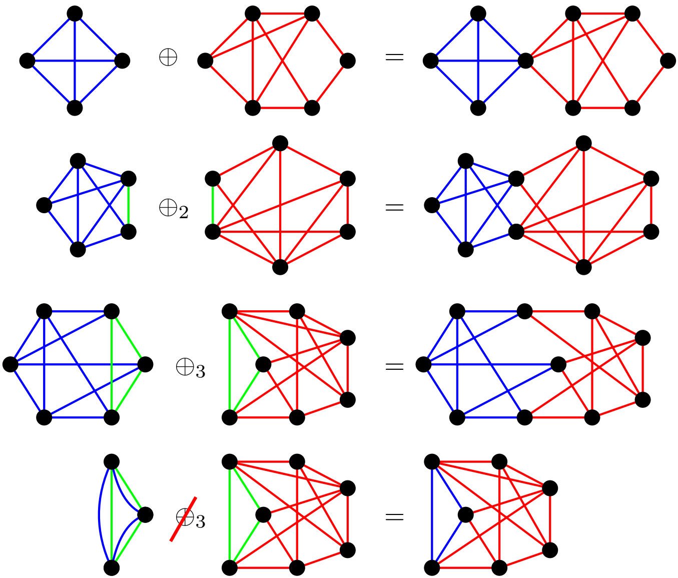

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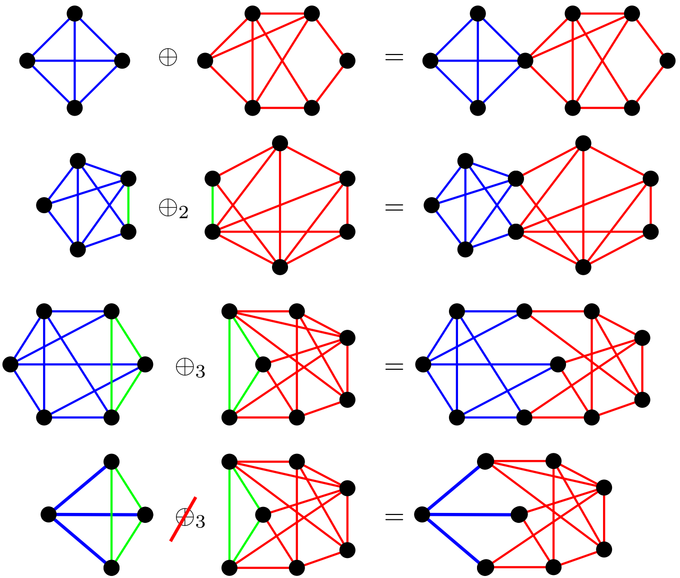

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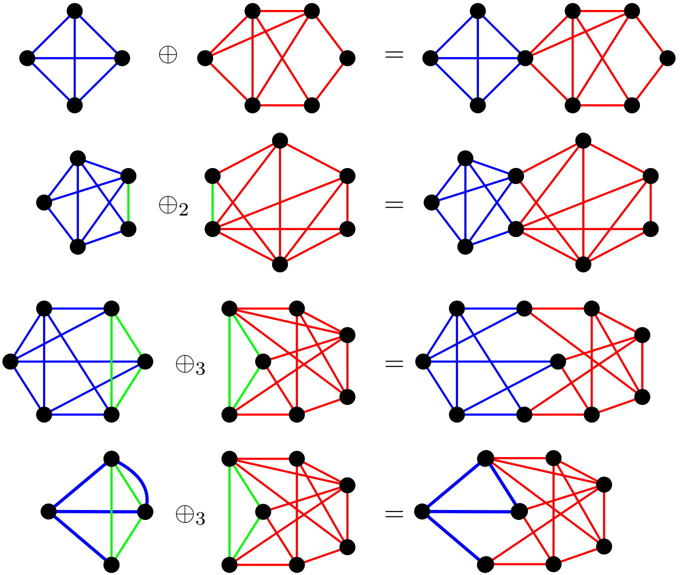
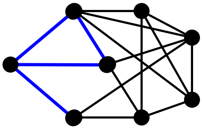


Connectivity

internally 4-connected violator, AKA 4-fan



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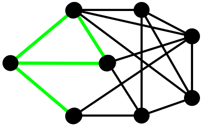

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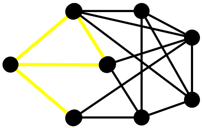

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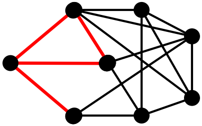

Internally 4-connected graphs

Theorem (2011)

Let G be an internally 4-connected graph. Then G has an internally 4-connected minor G' with

$$1 \leq |E(G) - E(G')| \leq 3$$

unless

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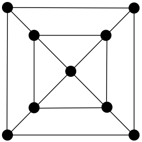
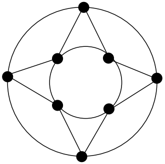
Let G be an internally 4-connected graph. Then G has an internally 4-connected minor G' with

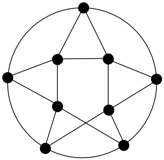
$$1 \leq |E(G) - E(G')| \leq 3$$

unless G or G^* is

(i) a terrahawk; or (ii) a planar or Möbius quartic ladder.



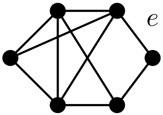




Splitter theorem

Theorem (Brylawski 1972, Seymour 1977)

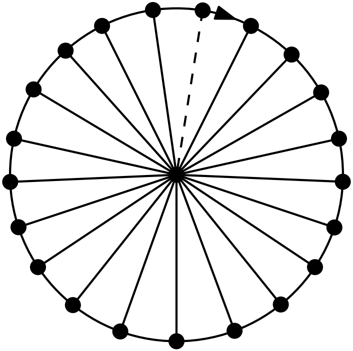
G, H 2-connected, $H \preceq G$, and $e \in E(G) - E(H) \implies G \backslash e$ or G/e is 2-connected with H as a minor



3-connected matroids

Theorem (Seymour's Splitter Theorem)

G & H 3-connected and $H \precneqq G$, where $|E(H)| \geq 4$ and, if H is a wheel, then G has no larger wheel minor $\implies$ $\exists G'$ where $G' \precneqq G$ and $H \preceq G'$ and $|E(G)| - |E(G')| = 1$



Internally 4-connected binary matroids

What do we know for graphs?
Johnson and Thomas (2001)

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What about binary matroids?
Geelen and Zhou (2006)
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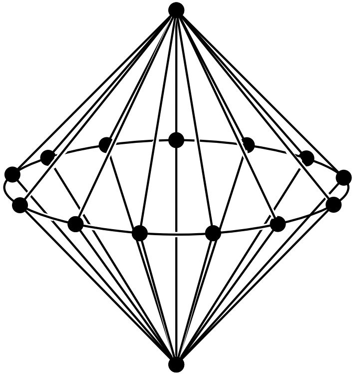
These allow the intermediate graph/matroid to satisfy some weaker form of connectivity.

Internally 4-connected graphs

How can we get an internally 4-connected minor of this?



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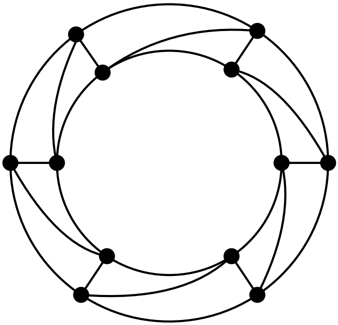

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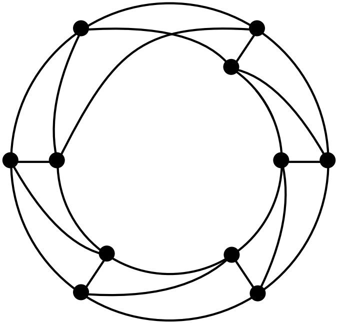

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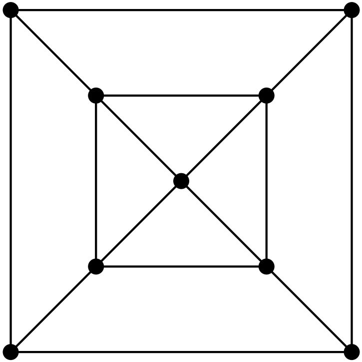

Internally 4-connected graphs

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G, H internally 4-connected graphs & $H \lneqq G \implies$
$\exists G'$, internally 4-connected, where $H \preceq G' \lneqq G$
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& $|E(G)| - |E(G')| \leq 3$ or, (up to duality),

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(i) $|E(G)| - |E(G')| = 4$ &

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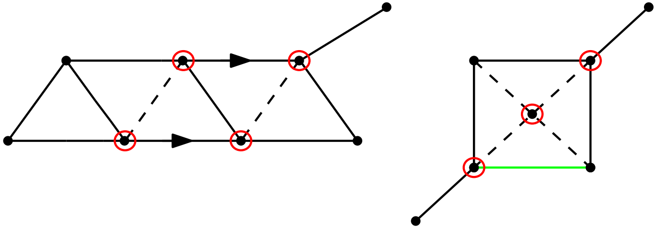

get G' from G by a central cocircuit deletion in a good augmented 4-wheel or a ladder-compression move; or ...

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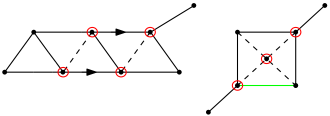
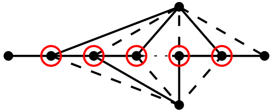


(ii) *get G' from G by trimming a double fan, a bowtie ring, or a "ladder;" or ...*



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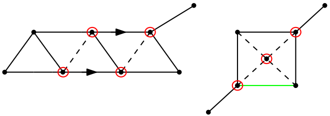

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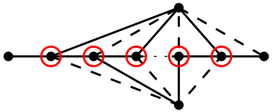

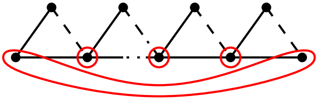

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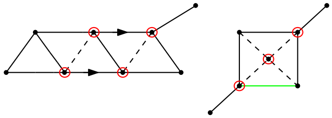

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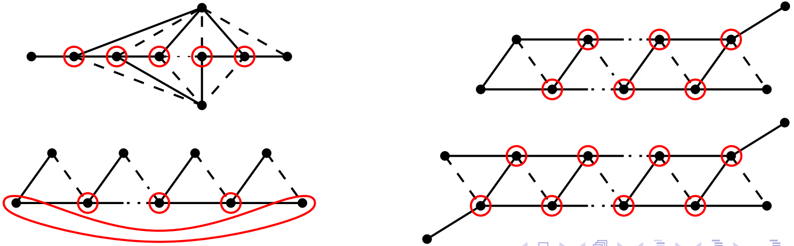

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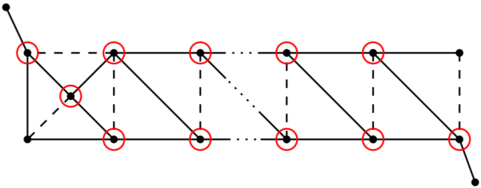

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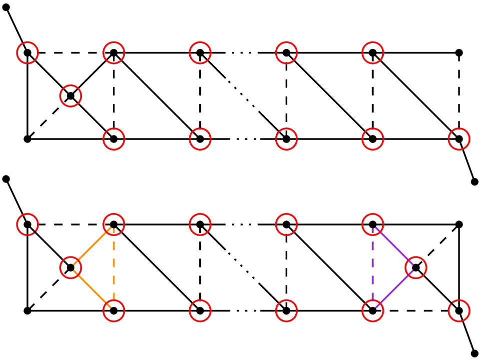

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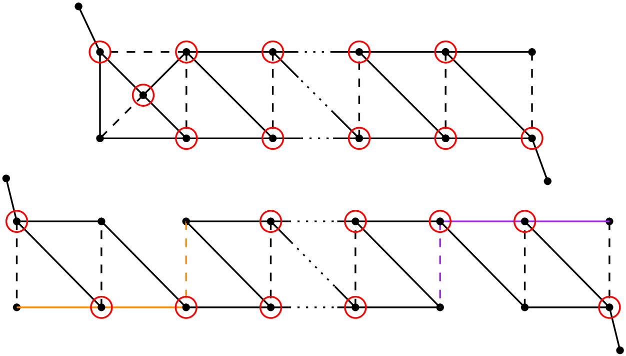

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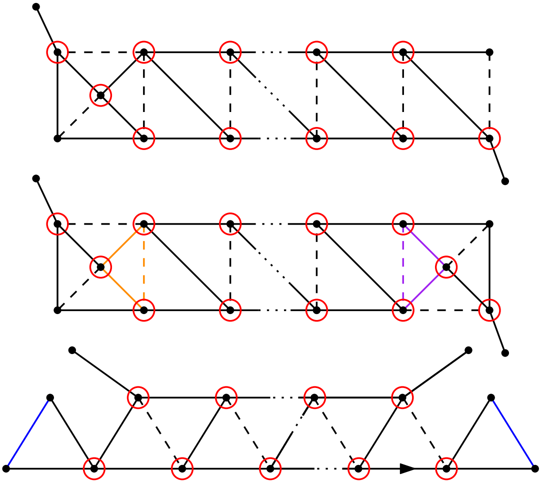

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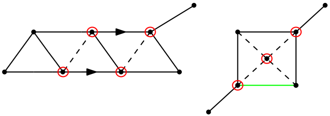

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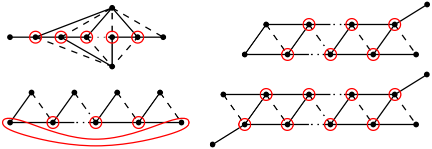

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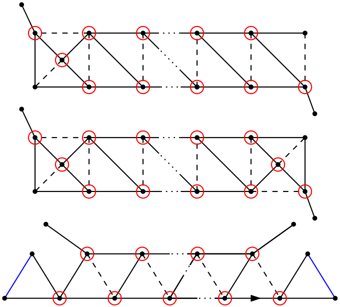
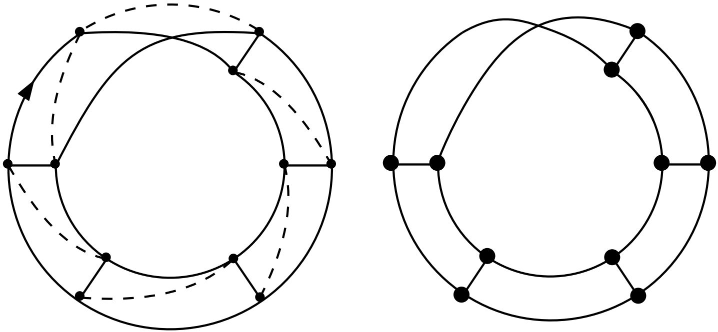


or . . .

Internally 4-connected graphs

(iv) (M, N) is (QML,CML); or . . .



Internally 4-connected graphs

(v) (G, H) is one of 12 *interesting pairs* (A splitter theorem for internally 4-connected binary matroids: small matroids, arXiv).

interesting pair (n.) $|E(G)| \leq 15$ and G and H are internally 4-connected, but no proper minor of G with a proper H-minor is internally 4-connected

Splitter Theorem for internally 4-connected graphs

Theorem

G, H internally 4-connected graphs & $H \lneqq G \implies$ $\exists G'$, internally 4-connected, where $H \preceq G' \lneqq G$ & $|E(G)| - |E(G')| \leq 3$ or G and H are Möbius ladders or comprise an interesting pair or, (up to duality), G' is obtained by one of these moves in G:

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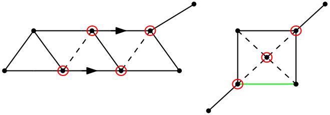

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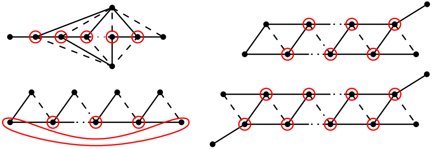

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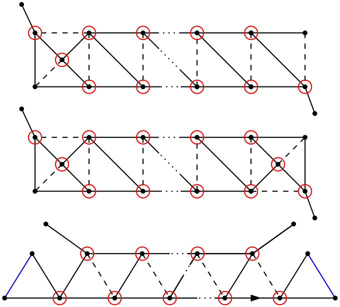

Splitter Theorem for internally 4-connected binary matroids

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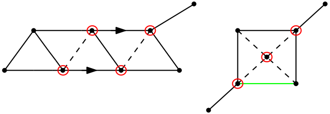

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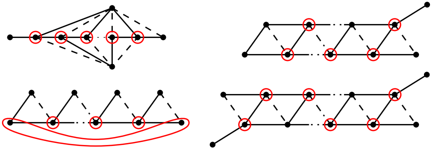

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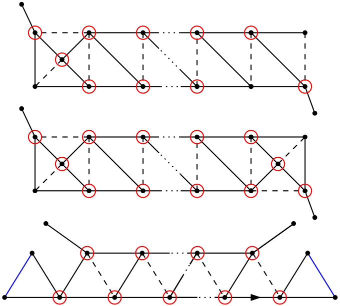

Corollary: internally 4-connected binary matroids

Theorem

M, N internally 4-connected binary matroids & $N \lneq M \implies \exists M'$, internally 4-connected, where $N \preceq M' \lneq M$ & $|E(M)| - |E(M')| \leq 6$ or M and N are Möbius ladders or Möbius matroids or, (up to duality), M' is obtained by one of these moves in M:

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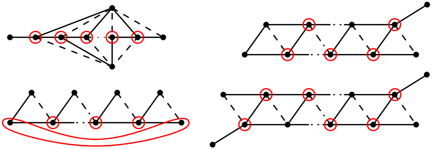

(ii)